Unsteady Stratified Fluid Flow over the Vertical Porous Plate in Presence of Hall Current, Radiation, Thermal Diffusion and Strong Magnetic Field

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Abstract—the studies of unsteady Stratified fluid flow over the vertical porous plate in presence of Hall current, Radiation, Thermal diffusion and strong Magnetic field have been studied numerically. This model is used for the laminar boundary layer flow of a stratified fluid. The system of governing equation is transformed into nonlinear ordinary coupled differential equations. These equations are solved numerically using the explicit finite difference method with the help of a computer programming language Compaq Visual Fortran 6.6a. Numerical results are obtained for the velocity, temperature and concentration distributions. The obtained results are presented graphically. Numerical solutions for velocity, temperature and concentrations distributions are obtained for the associated parameter. The stability conditions and convergence criteria of the explicit finite difference scheme are established.

Index Term:--Hall current; stratified fluid; thermal diffusion; explicit finite difference method; stability conditions; Radiation; convergence criteria;

1. Introduction

INternational researcher's attention is on the stratified fluid flow because of importance of natural convection flows of electrically-conducting fluids in the presence of transverse magnetic field. Due to the presence of some dissolved species stratified fluid flows have used in various industrial and engineering applications involving such examples nuclear reactors, metallurgical processes, crystal growth, heat exchangers, geothermal systems, ceramics industries, Environmental sciences, Chemical electrolytic reactors etc. Stratified fluids have been used in many scientific works like as Astrophysics, Geophysics, and Biomedical technology. The thermal stratification are realized when the fluid temperature increases with height in a gravitational field. A lot of free convection processes occur in environment with temperature stratification. The atmosphere is thermally stratified, for an example the ocean. Stratification of the fluid arises owing to a concentration differences, temperature variation, or the presence of different fluids like gas, oil, water, air etc. Thermal stratification is important in the sea, rivers, lakes, condensers of power plants and different industrial units.

The natural convection heat transfer from a non isothermal vertical flat plate immersed in a thermal stratified medium and similarity solutions have been carried out for a wide range of wall and ambient temperature distributions for a vide values of the Prandtl number between 0.1 and 20 studied by Yang et al. [1] . Srinivasan et al. [2] discussed the problem of two buoyancy-driving forces which are aid and oppose each other. A new class of similarity solutions for natural convection flow over an isothermal vertical wall in a thermally-stratified medium has been analysed by Kulkarni et al. [3]. Merkin [4] explored that a singular behavior is predicted for the case of non-isothermal surface with fixed environment temperature when a critical value of the wall temperature parameter is exceeded. A general new class of similar solutions for laminar natural convection boundary-layer flow along a heated vertical plate in a stratified environment has been reported by Henkes and Hoogendoorn [5]. The problem of steady laminar natural convection flow along a vertical permeable surface immersed in a thermally stratified environment in the presence of magnetic field and heat absorption effects has been numerically studied by Chamkha et ai. [6]. The transient natural convection of driven flow along an infinite vertical plate has been studied Shapiro et al. [7]. The two dimensional steady flow with heat and mass transfer effects along an isothermal vertical flat plate in a thermally stratified fluid. Boundary layer equations were solved by using an implicit finite difference method as well as local non-similarity method has been investigated Saha et al. [8]. Non-similarity solutions for the problem of steady, laminar, natural convective flow and heat transfer from a vertical circular cone immersed in a thermally stratified medium with either a uniform surface temperature or a uniform surface heat flux has been studied by Hossain et al. [9]. A study on the steady natural convection boundary layer flow over a continuously moving isothermal vertical surface immersed in thermally stratified medium has been presented by Takhar et al. [10]. Singh et al. [11] investigated integral method for free convection in thermally stratified porous medium. Sattar and Alam[12] investigated Thermal-diffusion as well as transportation effects on MHD free convection and mass transfer Flow past a Vertical porous plate.

Gansen et al. [13] reported viscous heating effects in doubly stratified free convection flow over vertical plate radiation and chemical reaction. MHD radiative heat and mass transfer nano-fluid flow past a horizontal stretching sheet in a rotating system has been studied by Hasan et al. [14]. M. M. Hasan [15] studied MHD heat and mass transfer stratified fluid flow through a vertical plate with viscous dissipation, radiation and thermal diffusion. The present study, therefore, unsteady Stratified fluid flow over the vertical porous plate in presence of Hall current, Radiation, Thermal diffusion and strong Magnetic field have been studied numerically. The obtained non-similar partial differential equations have been solved by explicit finite difference method [16-18].

2. Mathematical Model of Flow

Consider an MHD free convection heats and mass transfer flow of an electrically conducting viscous ,incompressible, stratified fluid flows through a vertical plate $y=0$.Considered the Cartesian coordinates x , measured along the plate surface and y is the coordinate measured normal to the plate surface and z is the cordinate normal to the plate. The flow is assumed to be in the x direction. At first, the fluid temperature and the plate temperature are same and it is T_∞ and concentration C_∞ . At times $t>0$, the temperature of the plate is raised to T_w and concentration near the plate is raised to C_w .For the ambient fluid, the temperature and concentration increase linearly with the

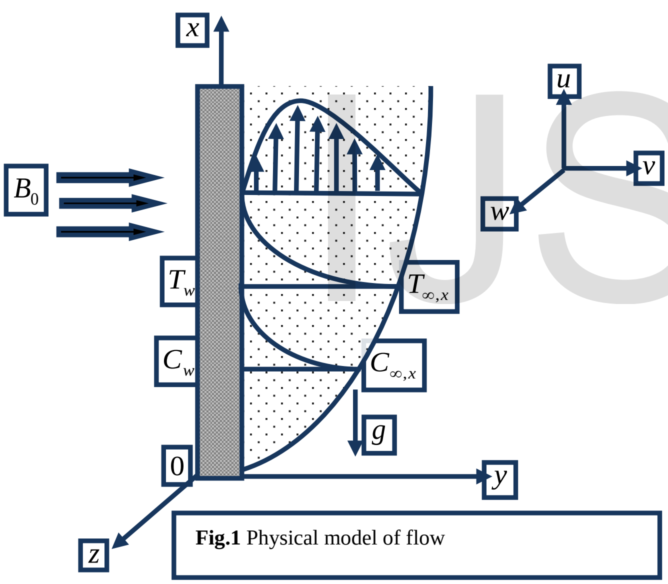

Fig.1 Physical model of flow

height, where $T_{\infty,0}$ and $C_{\infty,0}$ being the values at $x=0$. By using the above, statements, the governing boundary layer equations for the flow with Boussinesq's approximation are as follows.

The Continuity equation;

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

The Momentum equation in x-direction;

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\upsilon\left(\frac{\partial^2 u}{\partial y^2}\right)+g\beta_T\left(T-T_{\infty,x}\right)+g\beta_C\left(C-C_{\infty,x}\right)-\frac{\sigma B_0^2}{\rho\left(1+m^2\right)}(u+mw)-\frac{\mu}{\rho k_1}u \tag{2}$$

The Momentum equation in z-direction;

$$\frac{\partial w}{\partial t}+u\frac{\partial w}{\partial x}+v\frac{\partial w}{\partial y}=\upsilon\left(\frac{\partial^2 w}{\partial y^2}\right)+\frac{\sigma B_0^2}{\rho\left(1+m^2\right)}(mu-w)-\frac{\mu}{\rho k_1}w \tag{3}$$

The Energy equation;

$$\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{k}{\rho C_p}\left(\frac{\partial^2 T}{\partial y^2}\right)-\frac{1}{\rho C_p}\left(\frac{\partial q_r}{\partial y}\right)+\frac{\upsilon}{C_p}\left[\left(\frac{\partial u}{\partial y}\right)^2+\left(\frac{\partial w}{\partial y}\right)^2\right]-\gamma_T u \tag{4}$$

The Concentration equation;

$$\frac{\partial C}{\partial t}+u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D\left(\frac{\partial^2 C}{\partial y^2}\right)+\frac{Dk_t}{T_m}\left(\frac{\partial^2 T}{\partial y^2}\right)-\gamma_C u \tag{5}$$

The initial and boundary conditions are;

$$u=0, v=0, w=0,\ T=T_w,\quad C=C_w \quad \text{at } y=0 \quad \text{When } t>0 \tag{6}$$

$$u\to 0,\quad w\to 0,\quad T\to T_{\infty,x},\ C\to C_{\infty,x} \quad \text{as} \quad y\to\infty$$

Where $\gamma_T \equiv \frac{dT_{\infty,x}}{dx}+\frac{g}{C_p}$ and $\gamma_C \equiv \frac{dC_{\infty,x}}{dx}+\frac{g}{C_p}$, here $\frac{dT_{\infty,x}}{dx}$ represents the vertical thermal advection term and $\frac{dC_{\infty,x}}{dx}$ represents the vertical mass advectionterm. $\frac{g}{C_p}$ is the pressure work term known as compression.If the work of compression is very small then we can take $\gamma_T \equiv \frac{dT_{\infty,x}}{dx}$ and $\gamma_C \equiv \frac{dC_{\infty,x}}{dx}$.

The Roseland approximation [17] is expressed for radiative heat flux and leads to the form as,

$$q_r=-\frac{4\sigma^*}{3k^*}\frac{\partial T^4}{\partial y} \tag{7}$$

Where k^* is the mean absorption coefficient σ^* is the Stefan-Boltzmann constant [19]. The temperature difference with in the flow is sufficiently small. So that T^4 may be expressed as a linear function of the temperature, then the Taylor's series about T_∞ after neglecting higher order terms,

$$T^4 \cong 4TT_\infty^4-3T_\infty^3 \tag{8}$$

The dimensionless variables are as follows;

$$X=\frac{x}{L};\ Y=\frac{y}{L}G_r^{\frac{1}{4}};\ U=\frac{uL}{\upsilon}G_r^{\frac{-1}{2}};\ V=\frac{vL}{\upsilon}G_r^{\frac{-1}{4}};\ \overline{T}=\frac{\left(T-T_{\infty,x}\right)}{\left(T_w-T_{\infty,0}\right)};\ \overline{C}=\frac{\left(C-C_{\infty,x}\right)}{\left(C_w-C_{\infty,x}\right)};\ \tau=\frac{t\upsilon}{L^2}G_r^{\frac{1}{2}}$$

By using these non-dimensional quantities into the equations (1)-(5); we get the following dimensionless equations

$$\frac{\partial U}{\partial X}+\frac{\partial V}{\partial Y}=0 \tag{9}$$

$$\frac{\partial U}{\partial \tau}+U\frac{\partial U}{\partial X}+V\frac{\partial U}{\partial Y}=\frac{\partial^2 U}{\partial Y^2}+\overline{T}+B_f\overline{C}-\frac{M}{\left(1+m^2\right)}\left(U+mW\right)\text{ -}KU \tag{10}$$

$$\frac{\partial W}{\partial \tau}+U\frac{\partial W}{\partial X}+V\frac{\partial W}{\partial Y}=\frac{\partial^2 W}{\partial Y^2}+\frac{M}{\left(1+m^2\right)}(mU-W)-KW \tag{11}$$

$$\frac{\partial \overline{T}}{\partial \tau}+U\frac{\partial \overline{T}}{\partial X}+V\frac{\partial \overline{T}}{\partial Y}=\left(\frac{1+R}{P_r}\right)\frac{\partial^2 \overline{T}}{\partial Y^2}+E_c\left\{\left(\frac{\partial U}{\partial Y}\right)^2+\left(\frac{\partial W}{\partial Y}\right)^2\right\}-S_T U \tag{12}$$

$$\frac{\partial \overline{C}}{\partial \tau}+U\frac{\partial \overline{C}}{\partial X}+V\frac{\partial \overline{C}}{\partial Y}=\frac{1}{S_c}\frac{\partial^2 \overline{C}}{\partial Y^2}+S_r\frac{\partial^2 \overline{T}}{\partial Y^2}-S_M U \tag{13}$$

The corresponding boundary conditions are as;

$$\left.\begin{array}{l} U = 0, V = 0,\ W = 0,\ \overline{T} = 1,\ \overline{C} = 1 \text{ at } Y = 0 \\ U \to 0,\ W \to 0,\ \overline{T} \to 0,\ \overline{C} \to 0 \quad \text{as } Y \to \infty \end{array}\right\} \quad \text{When } \tau > 0 \tag{14}$$

The non-dimensional quantities are; $B_f = \dfrac{\beta_C\left(C - C_{\infty,x}\right)}{\beta_T\left(T - T_{\infty,0}\right)}$ (Bouncy frequency), $M = \dfrac{\sigma B_0^2 L^2}{\rho \upsilon} G_r^{\frac{-1}{2}}$ (Magnetic parameter), $P_r = \dfrac{\upsilon}{\alpha}$ (prandtl number), $R = \dfrac{16\sigma^* T_\infty^3}{3kk^*}$ (Radiation parameter), $E_c = \dfrac{\upsilon^2 G_r}{L^2 C_p\left(T_w - T_{\infty,0}\right)}$ (Eckert number), $S_T = \dfrac{\gamma_T L}{\left(T_w - T_{\infty,0}\right)}$ (Thermal stratification parameter), $S_c = \dfrac{\upsilon}{D}$ (Schmidt number), $S_r = \dfrac{Dk_t\left(T_w - T_{\infty,0}\right)}{T_m\left(C_w - C_{\infty,0}\right)}$ (Soret number), $S_M = \dfrac{\gamma_C L}{\left(C_w - C_{\infty,0}\right)}$ (Mass stratification parameter), $G_r = \dfrac{g\beta L^3\left(T_w - T_{\infty,0}\right)}{\upsilon^2}$ (Thermal Grashof number), $G_r = \dfrac{g\beta^* L^3\left(C_w - C_{\infty,0}\right)}{\upsilon^2}$ (Mass Grashof number), $K = \dfrac{\mu\upsilon}{\rho k_1 U^2}$ (permeability of the porous medium)

3. Numerical Solutions

To solve the non-dimensional system by the explicit finite difference method, it is required a set of finite difference equation. To obtain the difference equations the region of the flow is divided into a grid or mesh of lines parallel to X and Y axes where X -axis is taken along the plate and Y -axis is normal to the plate.

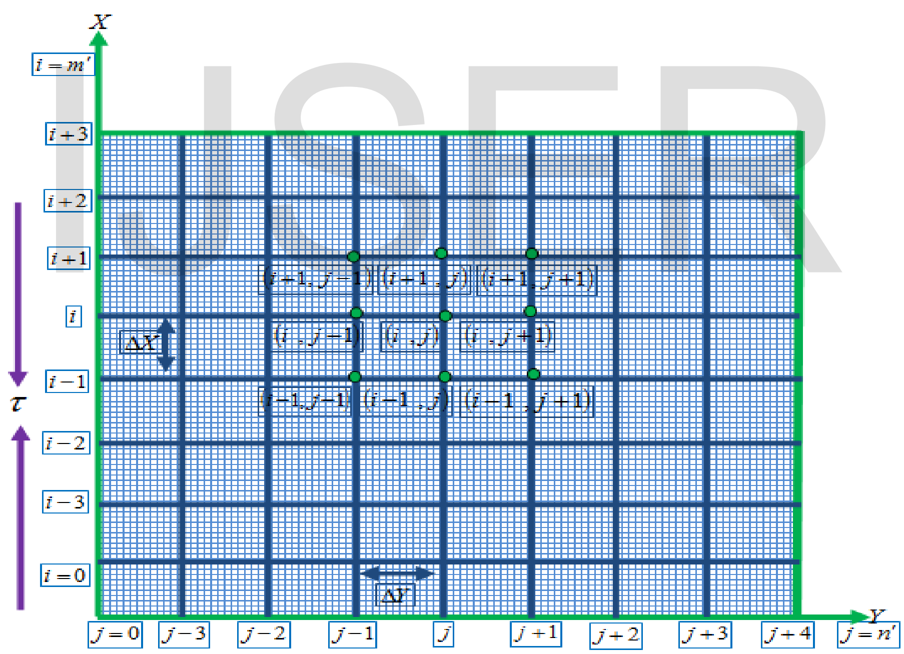

Fig. 2.Explicit finite difference system grid.

Here the plate of height $X_{\max}(=100)$ is measured i.e. X varies from 0 to 100 and assumed $Y_{\max}(=25)$ as corresponding to $Y \to \infty$ i.e. Y varies from 0 to 25 .There are $m(=200)$ and $n(=200)$ grid spacing in the X and Y directions respectively as shown Fig. 2. ΔX , ΔY are constant mesh size along X and Y directions respectively and taken as follows, $\Delta X = 1.00(0 \le X \le 100)$ and $\Delta Y = 0.25(0 \le Y \le 25)$ with the smaller time-step, $\Delta\tau = 0.005$ .Let U', W', $\overline{T}'$ and $\overline{C}'$ denote the values of U, W, $\overline{T}$ and $\overline{C}$ at the end of a time-step respectively. Using the explicit finite difference approximation, the system of partial differential equations (9)-(13) and the boundary conditions (14), an appropriate set of finite difference equations have been obtained as;

$$\frac{U'_{i,j}-U'_{i-1,j}}{\Delta X}+\frac{V'_{i,j}-V'_{i,j-1}}{\Delta Y}=0 \tag{15}$$

$$\frac{U'_{i,j}-U_{i,j}}{\Delta\tau}+U_{i,j}\frac{U_{i,j}-U_{i-1,j}}{\Delta X}+V_{i,j}\frac{U_{i,j+1}-U_{i,j}}{\Delta Y}=\frac{U_{i,j+1}-2U_{i,j}+U_{i,j-1}}{(\Delta Y)^2}+\overline{T}_{i,j}+B_f\overline{C}-\frac{M}{(1+m^2)}\left(U_{i,j}+mW_{i,j}\right)-KU_{i,j} \tag{16}$$

$$\frac{W'_{i,j}-W_{i,j}}{\Delta\tau}+U_{i,j}\frac{W_{i,j}-W_{i-1,j}}{\Delta X}+V_{i,j}\frac{W_{i,j+1}-W_{i,j}}{\Delta Y}=\frac{W_{i,j+1}-2W_{i,j}+W_{i,j-1}}{(\Delta Y)^2}+\frac{M}{(1+m^2)}\left(mU_{i,j}-W_{i,j}\right)-KW_{i,j} \tag{17}$$

$$\frac{\overline{T}'_{i,j}-\overline{T}_{i,j}}{\Delta\tau}+U_{i,j}\frac{\overline{T}_{i,j}-\overline{T}_{i-1,j}}{\Delta X}+V_{i,j}\frac{\overline{T}_{i,j+1}-\overline{T}_{i,j}}{\Delta Y}=\left(\frac{1+R}{P_r}\right)\left(\frac{\overline{T}_{i,j+1}-2\overline{T}_{i,j}+\overline{T}_{i,j-1}}{(\Delta Y)^2}\right)$$

$$+E_c\left\{\left(\frac{U_{i,j+1}-U_{i,j}}{\Delta Y}\right)^2+\left(\frac{W_{i,j+1}-W_{i,j}}{\Delta Y}\right)^2\right\}-S_TU_{i,j} \tag{18}$$

$$\frac{\overline{C}'_{i,j}-\overline{C}_{i,j}}{\Delta\tau}+U_{i,j}\frac{\overline{C}_{i,j}-\overline{C}_{i-1,j}}{\Delta X}+V_{i,j}\frac{\overline{C}_{i,j+1}-\overline{C}_{i,j}}{\Delta Y}=$$

$$\frac{1}{S_c}\left(\frac{\overline{C}_{i,j+1}-2\overline{C}_{i,j}+\overline{C}_{i,j-1}}{(\Delta Y)^2}\right)+S_r\left(\frac{\overline{T}_{i,j+1}-2\overline{T}_{i,j}+\overline{T}_{i,j-1}}{(\Delta Y)^2}\right)-S_MU_{i,j} \tag{19}$$

With initial and boundary conditions;

$$U^n_{i,0}=0,\ V^n_{i,0}=0,\ W^n_{i,0}=0,\ \overline{T}^n_{i,0}=1,\ \overline{C}^n_{i,0}=1 \tag{20}$$

$$U^n_{i,L}=0,\ V^n_{i,L}=0,\ W^n_{i,L}=0,\ \overline{T}^n_{i,L}=0,\ \overline{C}^n_{i,L}=0,\quad \text{where}\quad L\to\infty$$

Here the subscripts *i* and *j* designate the grid points with X and Y coordinates respectively and the subscript *n* represents a value of time, $\tau=n\Delta\tau$ where $n=0,1,2,3,\ldots\ldots\ldots$.

The stability conditions of the method are as follows;

$$U\frac{\Delta\tau}{\Delta X}+|-V|\frac{\Delta\tau}{\Delta Y}+2\frac{\Delta\tau}{(\Delta Y)^2}+M\frac{\Delta\tau}{2(1+m^2)}\le 1 \tag{21}$$

$$U\frac{\Delta\tau}{\Delta X}+|-V|\frac{\Delta\tau}{\Delta Y}+2\frac{\Delta\tau}{(\Delta Y)^2}\left(\frac{1+R}{P_r}\right)\le 1 \tag{22}$$

$$U\frac{\Delta\tau}{\Delta X}+|-V|\frac{\Delta\tau}{\Delta Y}+\frac{2}{S_c}\frac{\Delta\tau}{(\Delta Y)^2}\le 1 \tag{23}$$

Since from the initial condition, $U=V=\overline{T}=\overline{C}=0$ at $\tau=0$ and the consideration due to stability and convergence analysis is $E_c<1$ and $R\ge 0.10$ Hence convergence criteria of the method are $P_r\ge 0.73$, $m\ge 0.10$ and $S_c\ge 0.10$.

4. Results and Discussion

In order to investigate the physical significant of the problem, the numerical values of primary velocity, secondary velocity, temperature and concentration within the boundary layer have been computed for different values of various parameters.To obtain the steady-state solutions, the calculations have been carried out up to non-dimensional time $\tau=5$ to 80. It is observed that the numerical values of U, W, $\overline{T}$ and $\overline{C}$ however, show little changes after $\tau=10$. Hence at $\tau=10$ the solutions of all variables are steady-state solutions. The primary and secondary velocities are displayed in Figs. 3.for different values of Magnetic parameter. These results show that the primary and the secondary velocity are increasing with the increase of Magnetic parameter. In Figs.4.the primary and secondary velocity are illustrated for various values of Hall parameter. It is noted that the primary velocity is decreasing with the increase of Hall parameter while the secondary velocity is alsoincreasing near the plate. In Figs.5.the primary and secondary velocity are illustrated for various values of Bouncy frequency parameter. It is indicated that the primary and secondary velocity distribution are increasing with the increase of Bouncy frequency parameter.

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The temperature profile has been shown in Figs. 6-8 for various values of Radiation parameter, Eckert number, Mass stratification parameter, Thermal stratification parameter, and Schmidt number Soret number respectively. These results show that the fluid temperature distributions are increasing with the increase of Radiation parameter, Eckert number, Mass stratification parameter and Schmidt number. But opposite effect has been found with the increasing of Thermal stratification parameter and Soret number. In Figs.9-11 the concentration profile has been illustrated for various values of Radiation parameter, Eckert number, Mass stratification parameter, Thermal stratification parameter, Schmidt number, Soret number respectively. It is noted that the concentration distribution are decreasing with the increase of Radiation parameter, Eckert number, Mass stratification parameter, Schmidt number. But opposite effect has been found for Thermal stratification parameter and Soret number. In Figs.12.the fluid temperature distribution are increasing with the increase of Prandtl number while the reverse effect is observed in the concentration of the fluid.

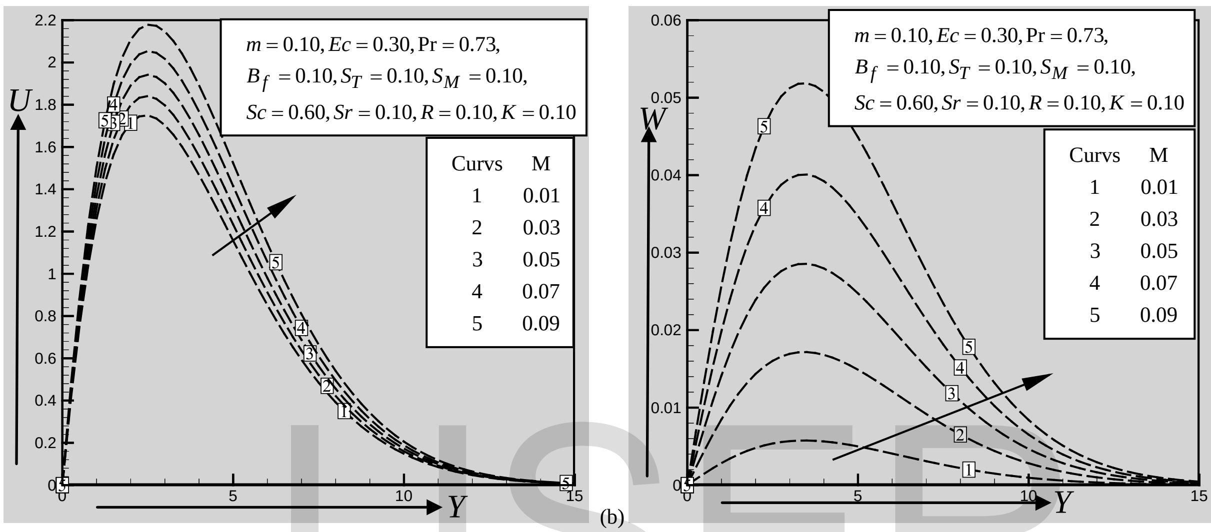

Fig.3. (a) Primary velocity profile and (b) Secondary velocity profile for different values of Magnetic parameter M .

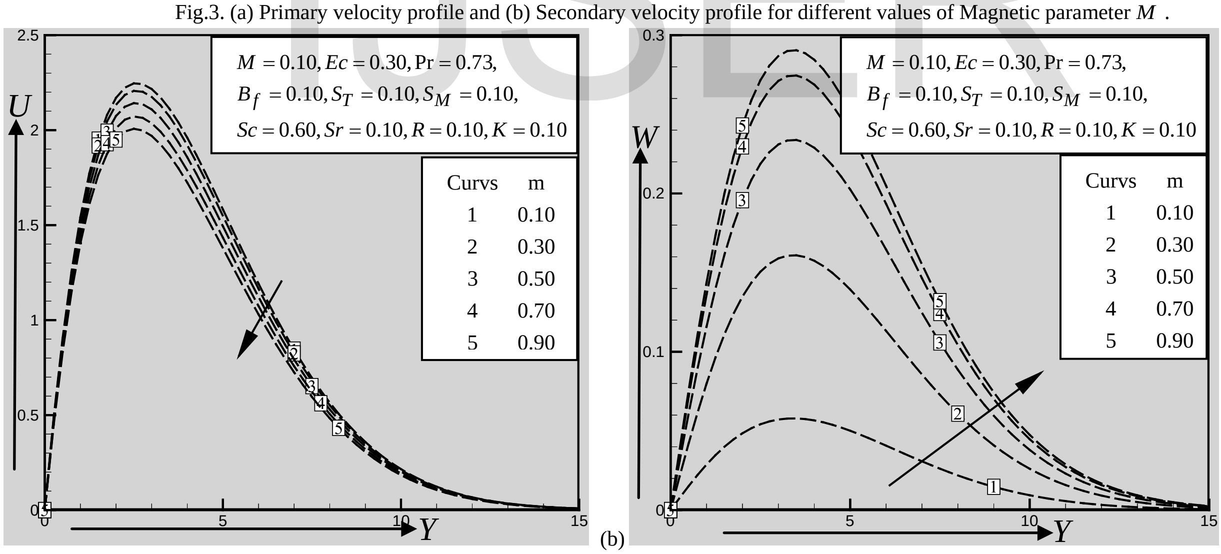

Fig.4. (a) Primary velocity profile and (b) Secondary velocity profile for different values of Hall parameter m .

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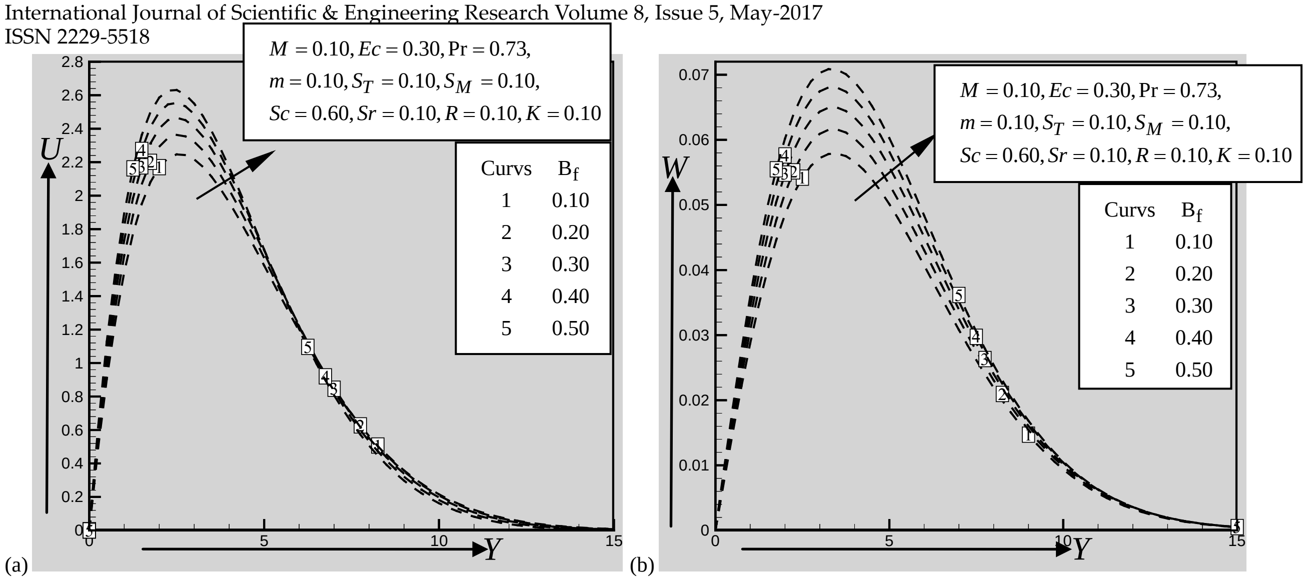

Fig.5. (a) Primary velocity profile and (b) Secondary velocity profile for different values of Bouncy frequency parameter B_f .

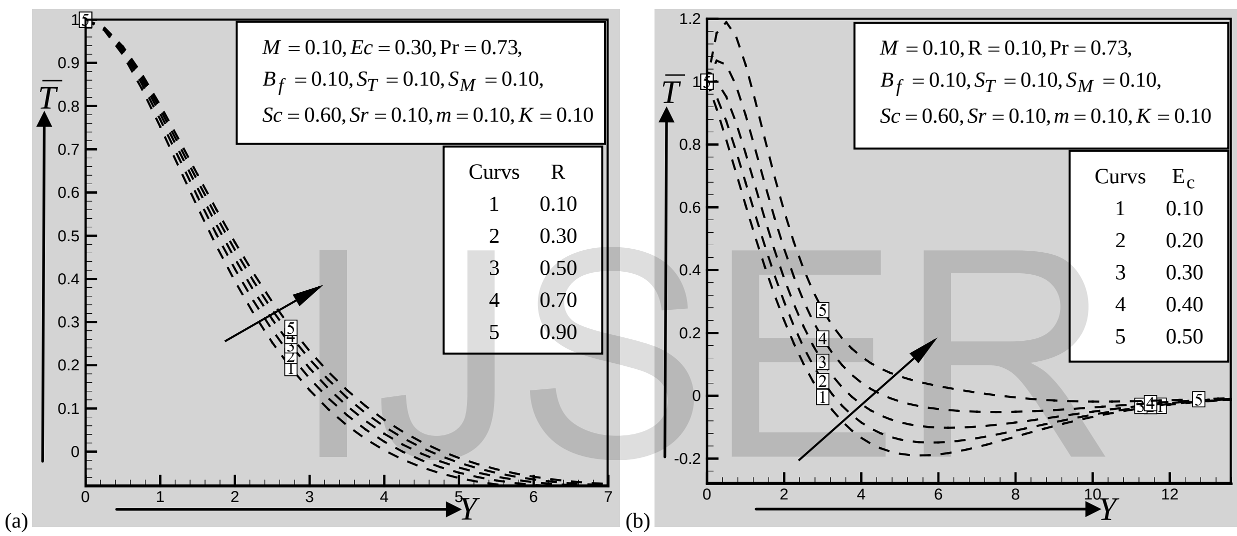

Fig.6.Temperature profile for different values of (a) Radiation parameter R ,(b)Eckert number E_c .

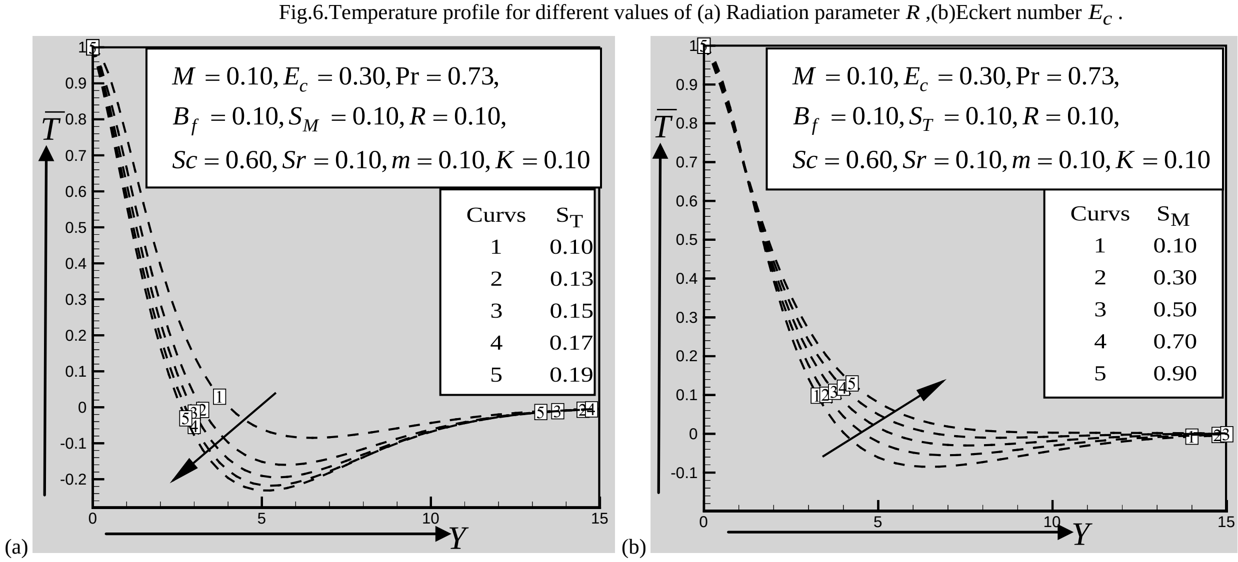

Fig.7.Temperature profile for different values of (a) Thermal stratification parameter S_T and (b) Mass stratification parameter S_M .

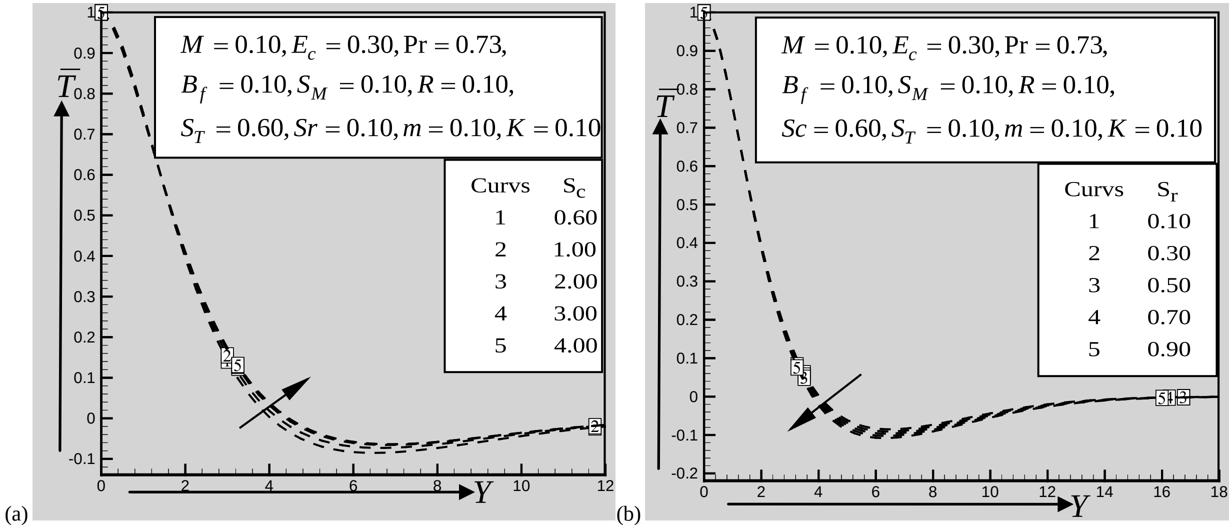

Fig.8.Temperature profile for different values of (a) Schmidt number S_c and (b) Soret number S_r .

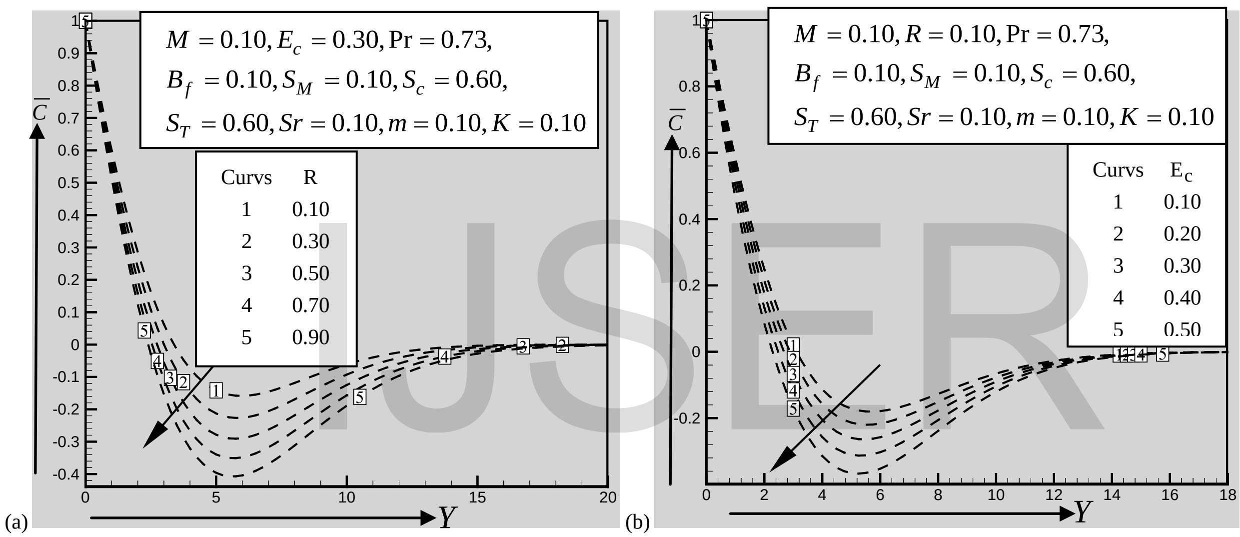

Fig.9.Concentration profile for different values of (a) Radiation parameter R ,(b)Eckert number E_c .

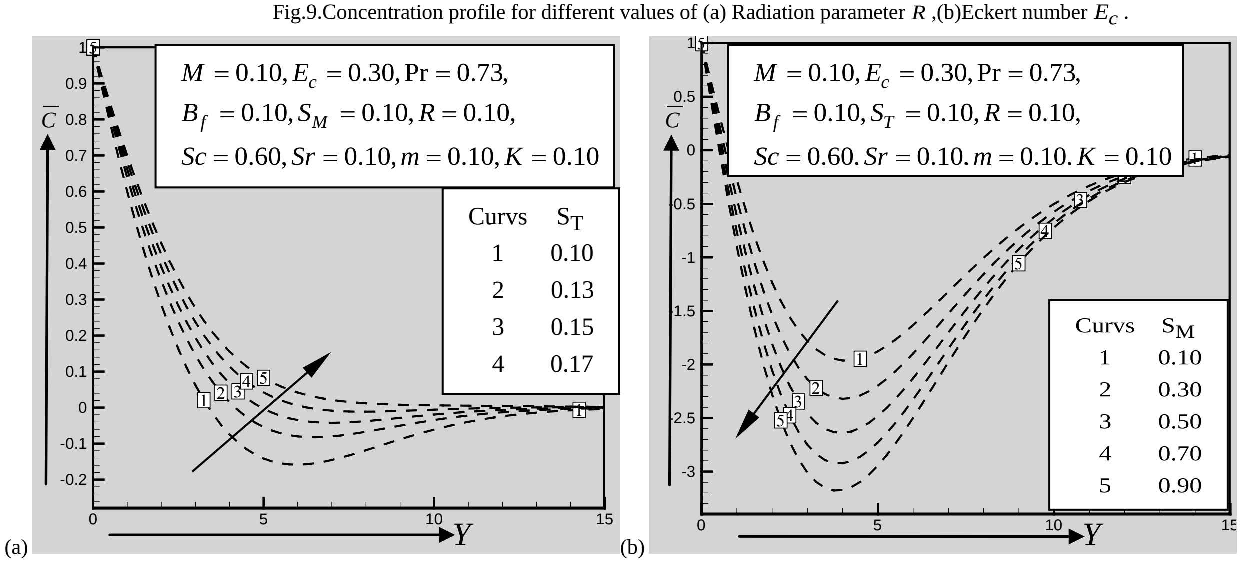

Fig.10.Concentration profile for different values of (a) Thermal stratification parameter S_T and (b) Mass stratification parameter S_M .

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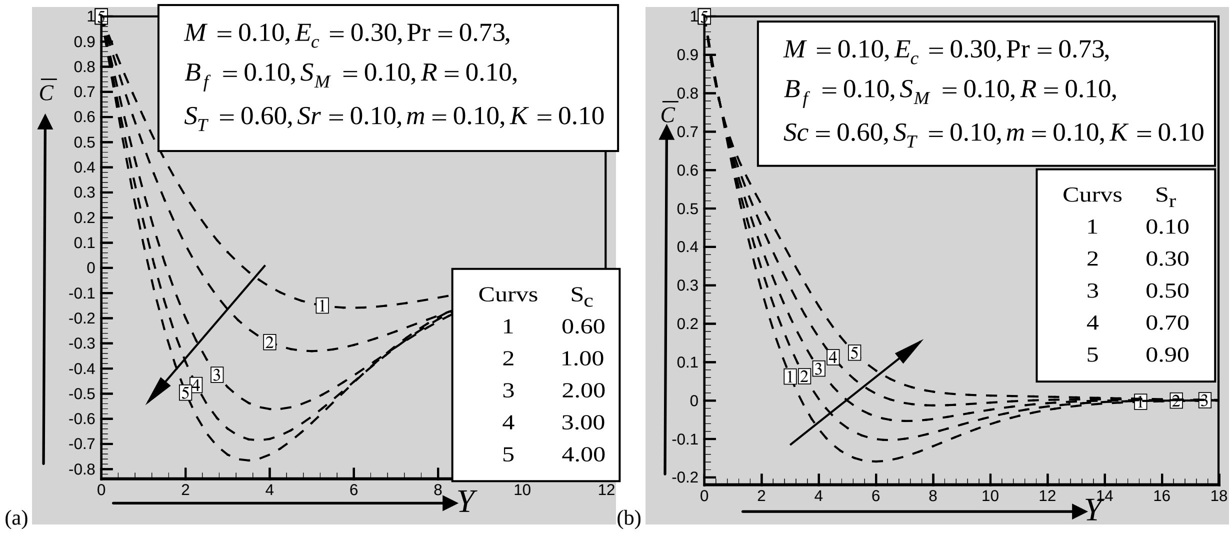

Fig.11.Concentration profile for different values of (a) Schmidt number S_c and (b) Soret number S_r .

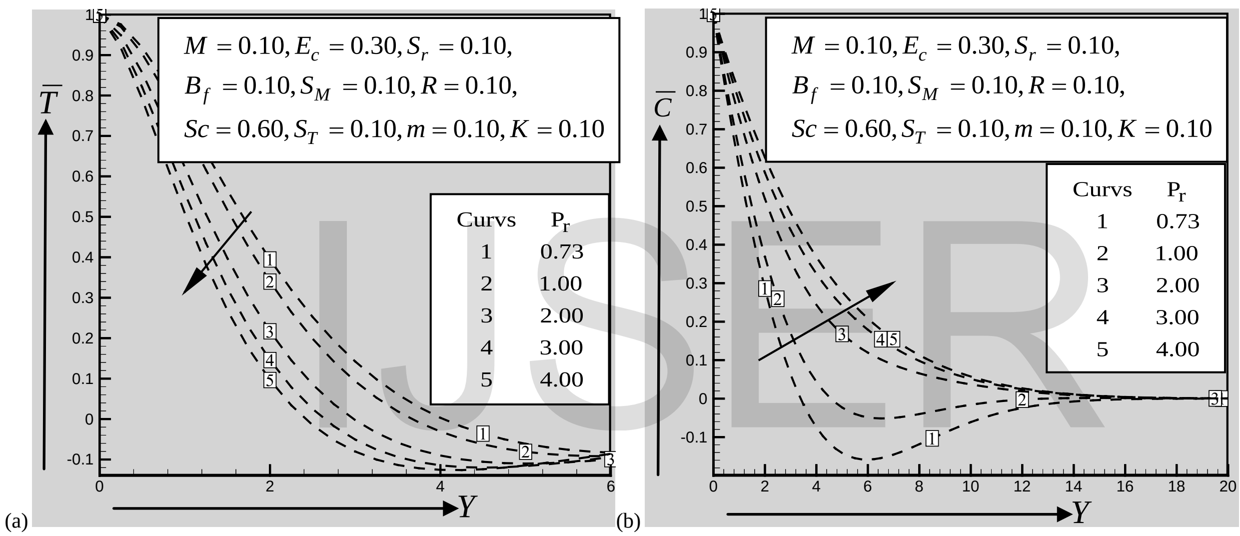

Fig.12.Temperature profile (a)and Concentration profile (b) for different values of Prandtl number P_r .

5. Conclusion

The finite difference of Numerical studies of unsteady Stratified fluid flow over the vertical porous plate in presence of Hall current, Radiation, Thermal diffusion and strong Magnetic field is investigated. The exactness of our outcome is qualitatively good in case of all parameters compare with the other published papers. Some significant findings of this study are specified below;

(i) For increasing the Magnetic and Bouncy frequency parameter, the primary and secondary velocity increases where as the primary velocity decreases and the secondary velocity decreases for increasing the Hall parameter.

(ii)The fluid temperature increases for increasing of Radiation parameter, Eckert number, Mass stratification parameter, Schmidt number, and decreases for increasing of Thermal stratification parameter, Soret number and Prandtl number.

(iii)The fluid concentration decreases for the increasing of Radiation parameter, Eckert number, Mass stratification parameter, Schmidt number and increases for increasing of Thermal stratification parameter, soret number and Prandtl number.

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